V5A3: ADVANCED TOPICS IN ALGEBRAIC GEOMETRY: INTRODUCTION TO RIGID GEOMETRY (2016 SPRING)

Rigid geometry was initiated by Tate in 1970s', and it can be viewed as analogue of classical complex geometry over non-archimedean fields (e.g. $\mathbb{Q}_p$). Now rigid geometry has become an important aspect in algebraic geometry, especially for its applications to number theory. In fact, many mathematical objects which have interesting arithmetic meanings are naturally described in terms of rigid geometry (e.g. eigenvarieties of p-adic modular forms, parameter spaces of Galois representations, ...). Moreover, rigid geometry can provide some new insights to some classical problems in algebraic geometry (whose formulation does not need rigid geoemtry), such as smooth compactification of the moduli spaces of abelian varieties, the Mumford-Manin conjecture, the Abyankar conjecture ...

Course Contents: The aim of this course is to give an introduction to basic knowledge of rigid geometry, so that students will have enough preliminaries for further reading of literature. I will basically follow Bosch's book *Lectures on formal and rigid geometry*, and the topics that I plan to cover are

(1) Non-archimedean fields, basic properties of Banach algebras over non-archimedean fields and Banach modules.
(2) Tate algebras and affinoid spaces.
(3) Tate's Acyclicity theorem and the Grothendieck topology on affinoid spaces.
(4) Rigid spaces and coherent sheaves on rigid spaces.
(5) The rigid GAGA-functor.
(6) Coherent cohomology on rigid spaces.
(7) The relationship with formal geometry: admissible formal schemes and admissible formal blow-up, rigid generic fiber of formal schemes, rigid points, construction of "good" formal models...
(8) Applications: Tate's elliptic curves. If time allows, I will also try to explain Mumford's rigid analytic uniformization of certain curves of genus ≥ 2.

Prerequisite: I will assume some familiarity in basic knowledge on commutative algebra (such as neotherian properties, flatness, completions), and also a first course on algebraic geometry (such as the basic notions of affine varieties, projective varieties, blow-up, coherent sheaves, coherent cohomology). For example, the materials in Atiyah and Macdanold's book *Introduction to commutative Algebra* and Chap. 1-3 of Harshorne's book are sufficient for this course.

References:

- S. Bosch, *Lectures on formal and rigid geometry*: this is somehow the main reference of this course, very comfortable to read.
- S. Bosch, U. Güntzer, R. Remmert, *Non-archimedean Analysis*: This book emphasizes the analytic point of view, and it contains much more materials than the course. It is better used as a reference for the proofs.
- A. Abbes, *Éléments de géométrie rigide: I*: This book is in some sense orthogonal to that of B.-G.-R., and it emphasizes the point of view of formal schemes. It is better used as a reference for things concerning formal models.

- J. Tate, Rigid analytic spaces, *Invent. Math.* 12, (1971), 257-289: This is the original paper where the notion of rigid analytic spaces was introduced, and also the so-called "Tate's acyclicity theorem" was first proved. Like many other papers by Tate, it is very well written. Since many terminologies in the paper were different from the standard ones used in nowadays, it is recommended to read perhaps after the course.