1. Write down (at least) five distinct classes of examples of Lagrangian submanifolds.

2. Let Q be a manifold and let $\eta \in \Omega^1(Q)$ be a 1-form. Check that $\operatorname{graph} \eta \subset T^*Q$ is Lagrangian iff η is closed, and is an exact Lagrangian iff η is exact.

3. Let $M = (M, \lambda)$ be an exact symplectic manifold. Prove or give a counterexample:

   (i) Symplectic isotopies (i.e. isotopies generated by a possibly time-dependent symplectic vector field) preserve exact Lagrangians.

   (ii) Hamiltonian isotopies (i.e. isotopies generated by a possibly time-dependent Hamiltonian vector field) preserve exact Lagrangians.

4. Prove or disprove: any symplectic manifold contains a closed Lagrangian submanifold.

5. Review the argument from class that an exact Lagrangian submanifold cannot bound a disk with positive symplectic area. In particular, it cannot bound a J-holomorphic disk for any J-compatible with the symplectic form.

   Remark. In contrast to the previous exercise, if (M, λ) is an exact symplectic manifold, it can happen that it contains no closed *exact* Lagrangian submanifolds. One example, which goes back to Gromov, is $(\mathbb{R}^{2n}, \lambda = \sum_i x_i dy_i - y_i dx_i$. The idea of the proof is to argue that such a Lagrangian submanifold, if it existed, would bound a holomorphic disk, which is impossible since the Lagrangian is exact.

6. Recall that the *Nearby Lagrangian conjecture* says that every closed exact Lagrangian submanifold of T^*Q is Hamiltonian isotopic to the zero section. Prove the Nearby Lagrangian conjecture for T^*S^1.

7. Prove that any closed Lagrangian in $(\mathbb{R}^{2n}, \omega_0)$ has vanishing Euler characteristic. (I stated this in class and sketched the proof – your task is to got through the argument and fill in the details).