Minisymposium 20 - Nonlinear and Stochastic Optimization

Elliptic Optimal Control Problems with Mixed Constraints

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In this talk we consider the following class of linear-quadratic optimal control problems with state y and control u:

$$(\mathbf{P}(\delta)) \quad \text{Minimize } \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u - u_d\|^2_{L^2(\Omega)} - \int_\Omega y\,\delta_1\,dx - \int_\Omega u\,\delta_2\,dx$$

subject to $u \in L^2(\Omega)$ and the elliptic state equation

$$\begin{aligned} -\Delta y &= u + \delta_3 \quad \text{on } \Omega \\ y &= 0 \quad \text{on } \partial\Omega \end{aligned} \tag{1}$$

as well as pointwise pure and mixed control-state constraints

$$\begin{aligned} u - \delta_4 &\geq 0 \quad \text{on } \Omega \\ \varepsilon u + y - \delta_5 &\geq y_c \quad \text{on } \Omega. \end{aligned} \tag{2}$$

Problem $(\mathbf{P}(\delta))$ depends on a parameter $\delta = (\delta_1, \delta_2, \delta_3, \delta_4, \delta_5)$, and we prove the Lipschitz stability of the unique optimal solution in $L^\infty(\Omega)$, with respect to perturbations in δ. The presence of simultaneous control and mixed constraints (2) requires a refinement of previously used techniques.