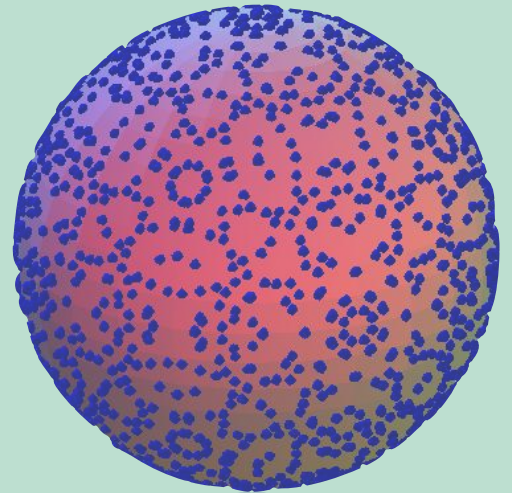


## Learning Seminar in Number Theory

# Equidistribution: the modern ergodic method



Let  $d > 1$  be a squarefree integer and let  $\mathcal{H}_d$  be the set of integer points on a 2-dimensional sphere of radius  $\sqrt{d}$ . Classical number theory shows that the number of such points is related to the class number of  $\mathbb{Q}(\sqrt{-d})$  and thus grows roughly like  $\sqrt{d}$ . One then naturally asks: as  $\mathcal{H}_d$  gets large, how is it distributed on the sphere? The study of this question turns out to be extremely rich, deeply connected with the theory of modular  $L$ -functions and the theory of homogeneous dynamics.

Using his “ergodic method”, Y. V. Linnik showed in the 50’s that  $\mathcal{H}_d$  (suitably normalised) becomes equidistributed on the sphere with respect to the Lebesgue measure as  $d \rightarrow \infty$  and satisfying the condition  $d \equiv \pm 1 \pmod{5}$ . Thirty years later, W. Duke removed the congruence condition using entirely different methods, based on harmonic analysis and the theory of  $L$ -functions.

In this seminar, we intend to read and compare two beautiful papers giving a modern interpretation of Linnik’s method and proposing new results and directions:

- M. Einsiedler, E. Lindenstrauss, P. Michel, A. Venkatesh, The distribution of closed geodesics on the modular surface and Duke’s theorem, *Einseign. Math* (2) 58 (2012)
- J. S. Ellenberg, P. Michel, A. Venkatesh, Linnik’s ergodic method and the distribution of integer points on spheres, *Tata Inst. Fundam. Res. Stud. Math.* 22 (2013) [also the source of the sphere picture above]

Depending on the preference of the seminar participants, we hope to continue by looking into a new generation of problems falling under the topic of *joint equidistribution*, in the spirit of the Michel-Venkatesh conjecture.

**Format:** weekly talks of 1,5 hours given by the participants. We will have an introductory talk in the first meeting and then distribute the other talks.

### Organisers:

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**MPIM Little Seminar Room**  
**Tuesdays, 14:15 - 15:45**  
**starting October 10th**